Lines, Curves, and Graphs

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My project “Lines, Curves and Graphs” was based on the study of linear and nonlinear relationships. Students completed their study of the units *Moving Straight Ahead, Thinking with Mathematical Models,* and *Growing, Growing, Growing*. Throughout these three units, five classes of students used the SMART Board™ interactive whiteboard in conjunction with graphing calculators and computers to introduce, enforce and strengthen their understanding of graphing linear relationships, nonlinear relationships and exponential functions.

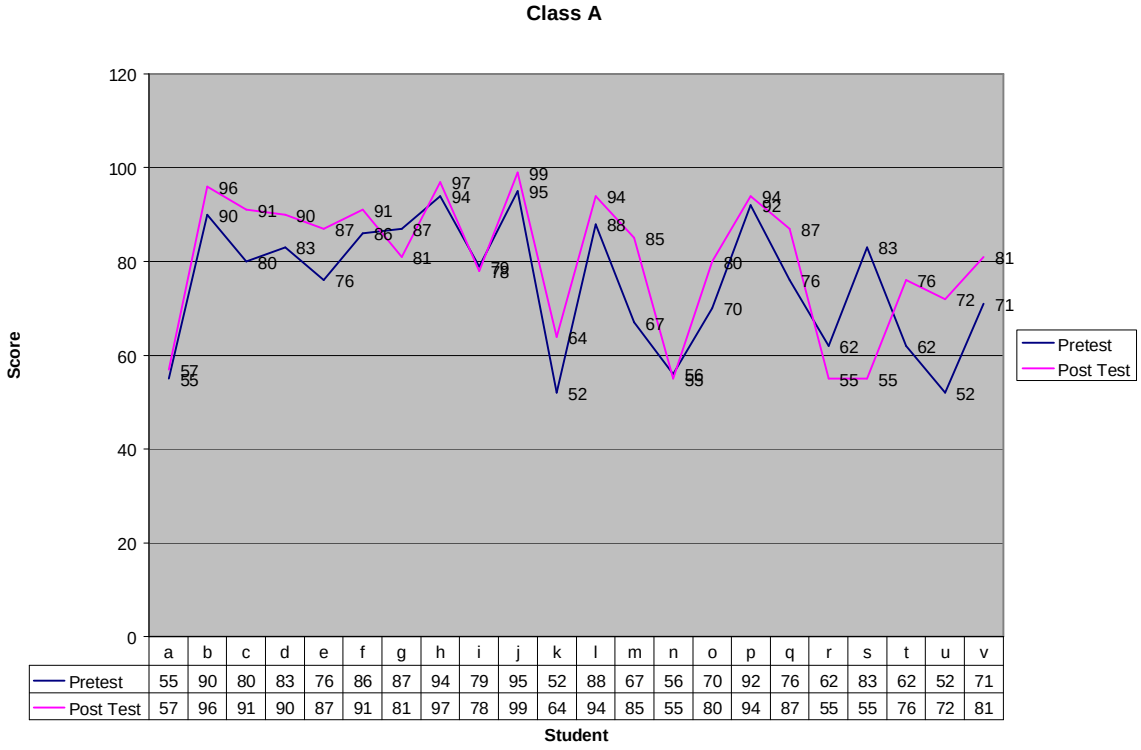
At our school, students are heterogeneously mixed into one of five math classes. Each class consisted of a wide range of abilities that included both special education students and basic skills students. The math program used for this study and for our curriculum is *Connected Mathematics* by Prentice Hall. Each class period was 43 minutes in length, and most classes had two teachers that co-taught lessons.

Our first unit, *Moving Straight Ahead,* focused on the study of linear relationships. Students compared two variables that had a constant rate of change. Through their discovery, students measured variables, such as time versus distance, time versus walking rates, and the distance of the rise of a step to the run of a step. Using these variables, students found the slopes and y intercepts by using graphs, tables, and equations. Students showed that using TI Interactive on both the SMART Board and the computers, helped to increase their visual understanding of linear relationships. Students were able to input information into excel, TI interactive and SMART Notebook to illustrate their data.

One experiment that students used to demonstrate linear relationships was “Climbing Monkeys.” This experiment, which is found at the following link, http://www.phschool.com/atschool/cmp2/active_math/site/Grade7/monk/index.html, demonstrated the constant rate of change between the distance the monkeys climbed up or down a palm tree. The website was displayed on the SMART Board unit and students changed the variables being tested. Students saw how the linear relationships changed when one or more of the variables being tested changed. The original position of the green monkey was 0 and the blue monkey was 100. By looking at the graph, students saw that the green monkey started at the origin and the blue monkey started at the top of the graph. As students changed the original position, they saw on the SMART Board interactive whiteboard, that the original position moved on the Y axis.

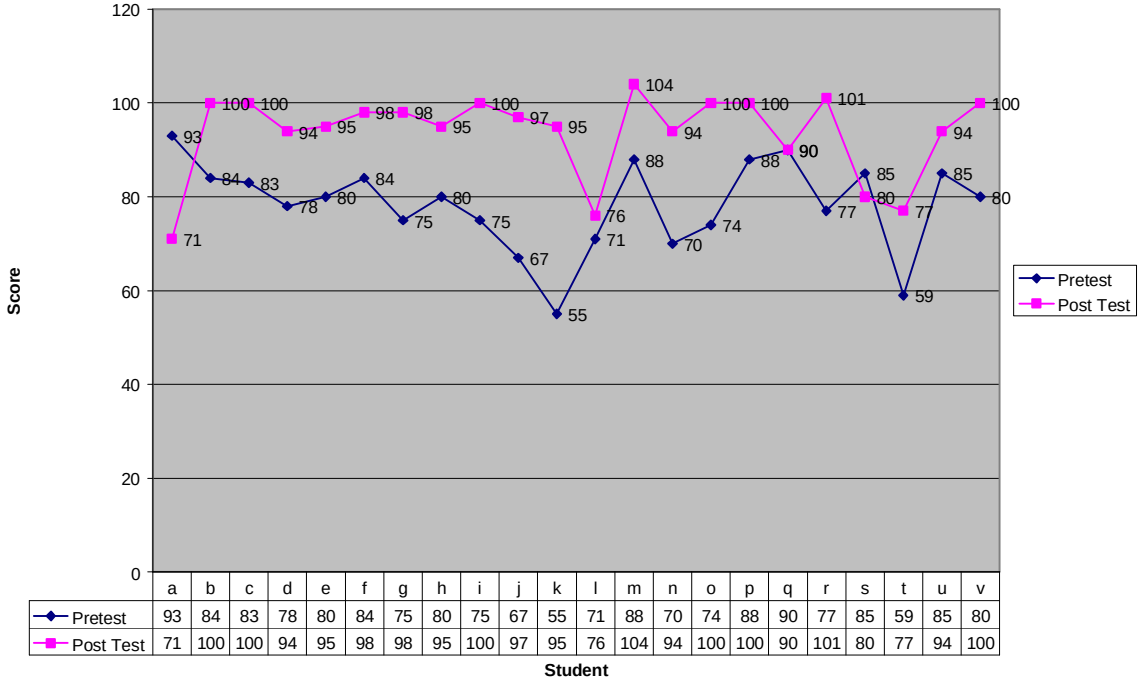
Students then changed the reach of each monkey and noticed that it affected the steepness of the line, as well as the total amount of time it took to climb up or down the palm tree. Students determined that each monkey still moved at a constant rate but as the variables changed, so did the Y intercept and the steepness of each line.

By using “Climbing Monkeys” on the SMART Board interactive whiteboard, students could visually identify the properties of a linear relationship. Although “Climbing Monkeys” was only one of the experiments conducted in *Moving Straight Ahead,* it was the most visual activity to introduce and reinforce linear relationships to the eighth-graders. The posttest was given at the end of the unit and you can see in the tables and graphs on pages 3 through 6 for *Moving Straight Ahead*, all classes increased their average score from the pretest to the posttest. When comparing the data from each class separately, I found that most students increased their scores from the pretest to the posttest. The results of their pretest versus the posttest showed an average increase of 6.462049 points, which in most cases increased the students’ scores by at least one grade level, over five class periods. The graphs and tables on the next two pages illustrate the average increase per class, as well as the progress each student made for their class period.



	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v
Pretest	55	90	80	83	76	86	87	94	79	95	52	88	67	56	70	92	76	62	83	62	52	71
Post Test	57	96	91	90	87	91	81	97	78	99	64	94	85	55	80	94	87	55	55	76	72	81

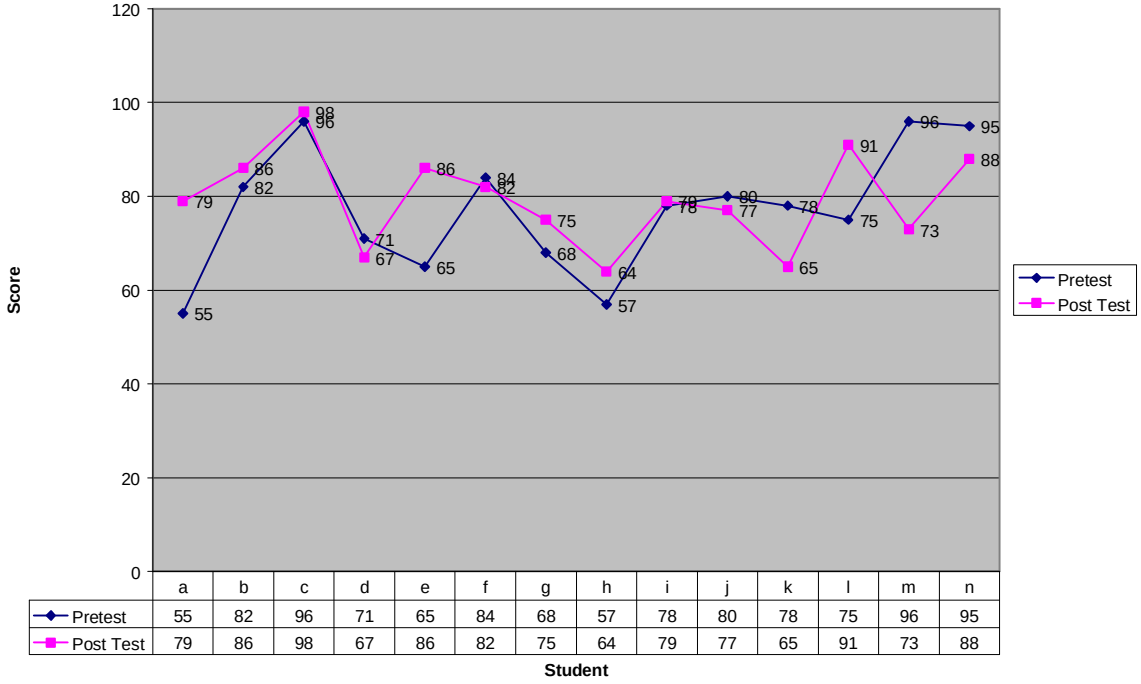
Class B



	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v
Pretest	93	84	83	78	80	84	75	80	75	67	55	71	88	70	74	88	90	77	85	59	85	80
Post Test	71	100	100	94	95	98	98	95	100	97	95	76	104	94	100	100	90	101	80	77	94	100

Student

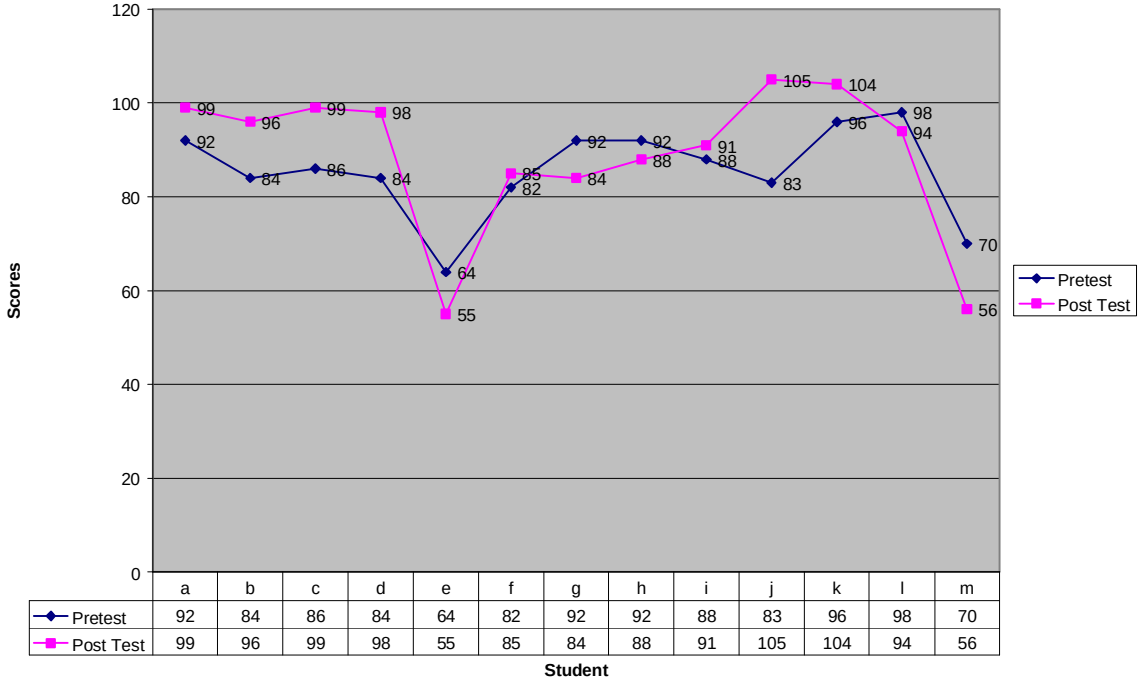
Class C



	a	b	c	d	e	f	g	h	i	j	k	l	m	n
Pretest	55	82	96	71	65	84	68	57	78	80	78	75	96	95
Post Test	79	86	98	67	86	82	75	64	79	77	65	91	73	88

Student

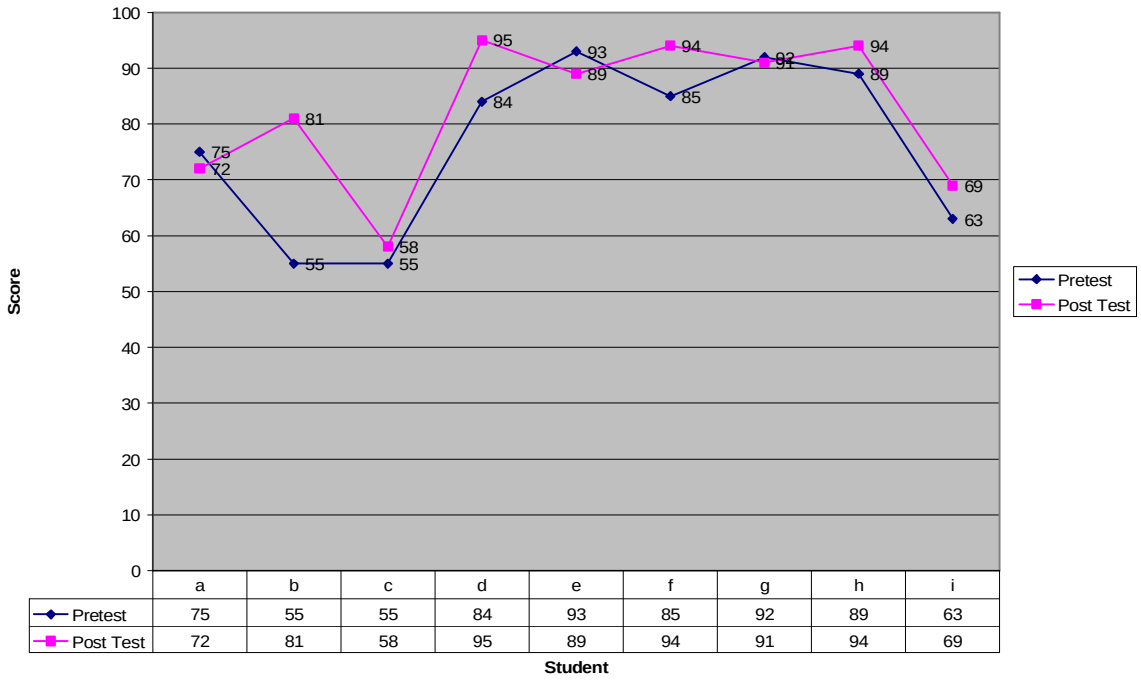
Class D



	a	b	c	d	e	f	g	h	i	j	k	l	m
Pretest	92	84	86	84	64	82	92	92	88	83	96	98	70
Post Test	99	96	99	98	55	85	84	88	91	105	104	94	56

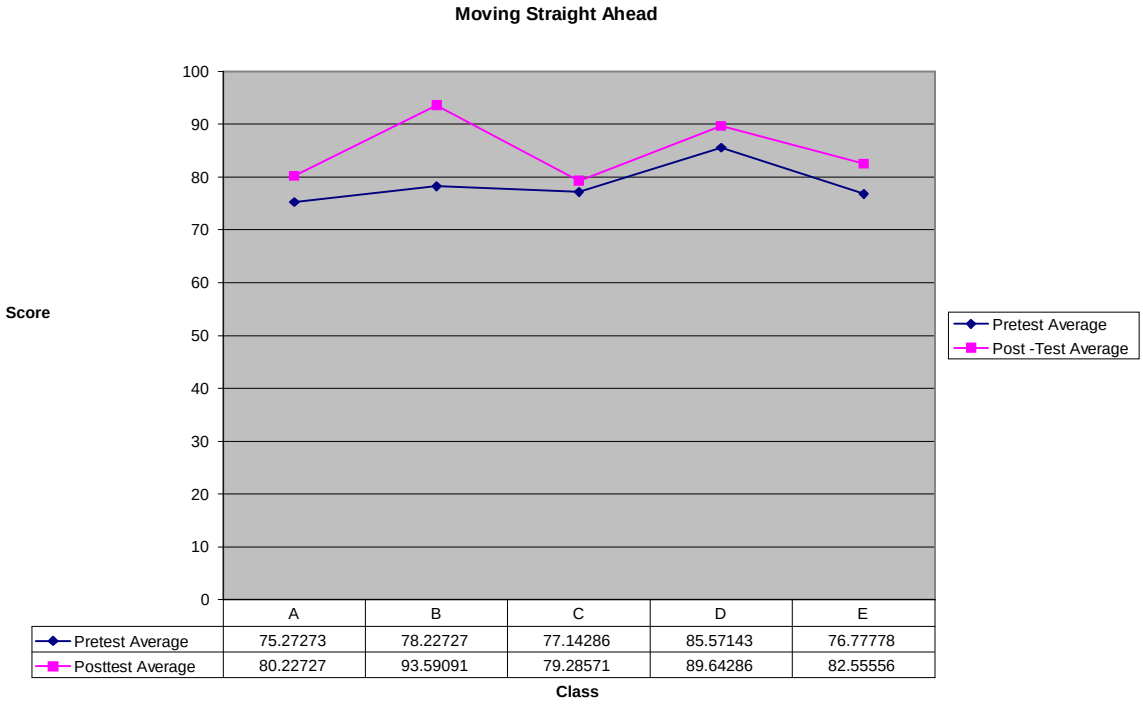
Student

Class E



	a	b	c	d	e	f	g	h	i
Pretest	75	55	55	84	93	85	92	89	63
Post Test	72	81	58	95	89	94	91	94	69

Student



The second unit of study for "Lines, Curves, and Graphs" was *Thinking with Mathematical Models.* This unit focused on both linear and nonlinear relationships. Students were able to describe characteristics of linear relationships by creating graphs on the SMART Board interactive whiteboard that showed a constant rate of change that were increasing, decreasing, or staying the same. By using TI Interactive™ computer software, students created what they believed were linear relationships. Once students created the graphs, they were able to check tables for each graph to see if the rate of change was constant between all of the points and determine if the line was linear.

To reinforce linear relationships, students used the activity "Virtual Bridge Experiment" at http://www.phschool.com/atschool/cmp2/active_math/site/Grade8/Bridge/index.html to compare the bridge's thickness versus its breaking weight. To begin, students set the length of the bridge at five and changed the thickness of the bridge. They noticed that as the thickness of the bridge increased by one, the weight increased by an average of four pennies. Students recorded the data in a table in Excel® spreadsheet software and graphed their table to determine if there was a linear relationship between the thickness of the bridge and the number of pennies used to break the

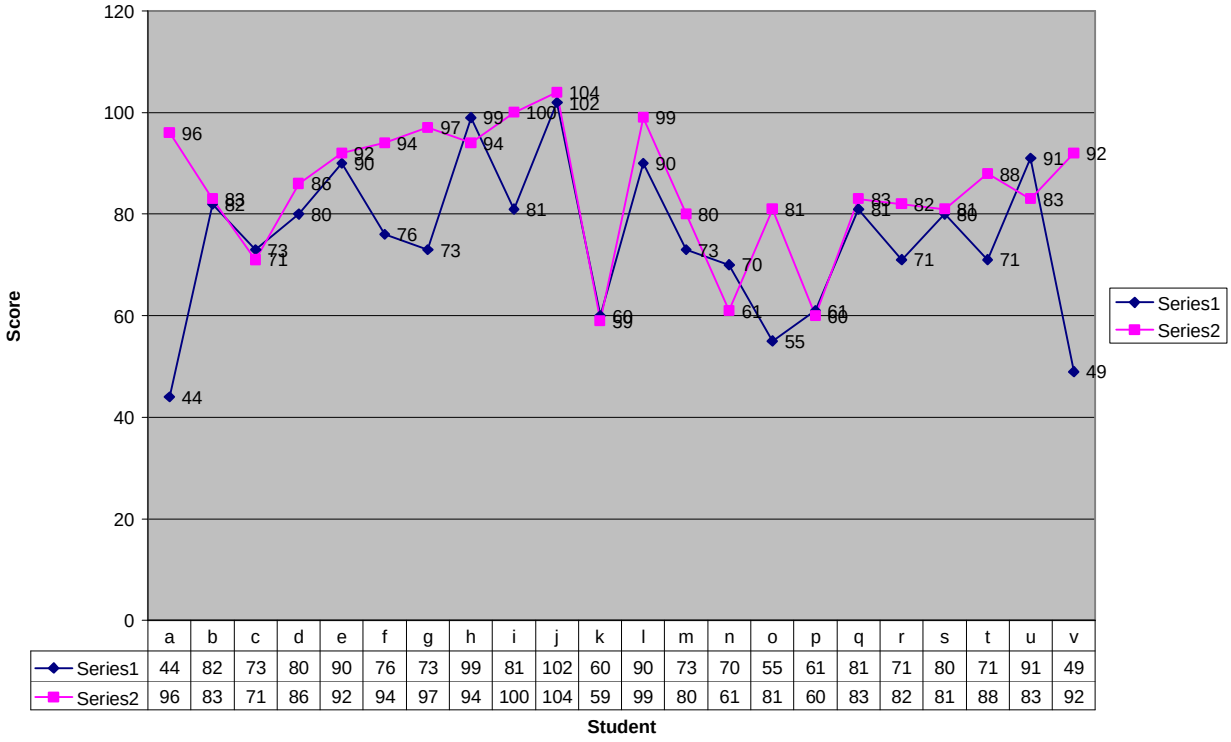
bridge. From their graph, students concluded that there was, indeed, a constant rate of change between the two variables being tested.

Students also explored nonlinear relationships in this unit. Students, again, used the "Virtual Bridge Experiment" to compare the bridge length versus the breaking weight. This time, students set the bridge thickness at one and changed the length. Students noticed the shorter the bridge, the more pennies the bridge could hold but decrease in the number of pennies was no longer constant. Students, again, recorded their data into an excel spreadsheet and graphed their table. This time, they found that the line was no longer linear but created something like a curve, or in their terms, a slide. By looking at this graph and table, students determined properties of two variables that do not have a constant rate of change and do not create a linear relationship. In order to strengthen the idea of nonlinear relationships, students also explored the concept of interest.

To thoroughly understand the concept of interest, each student was given $100 of play money. Students had the right to deposit the money into a play account at a daily interest rate of 1.5%. Students could withdraw the money at any time. Students could use the money to buy extra credit points, homework passes, computer time, or other classroom activities. Each activity had a price. During the first five minutes of class time, students determined the amount of play money in their account and plotted it on the SMART Board interactive whiteboard. Some of the students watched their money increase rapidly, while others watched their money decrease rapidly. This process showed students that their increase and decrease changed over time but the increase or decrease was not a steady rate of change.

In both the pretest and posttest for *Thinking with Mathematical Models,* students were given 25 multiple-choice questions and two open-ended questions. Students used graph paper and graphing calculators to complete the pretest but were given the opportunity to use their data from the "Virtual Bridge Experiment" and the interest project to help them with the posttest. Below are the results from both the pretest and the posttest for each of the five classes, as well as the class averages. Students inputted their data into Excel spreadsheet software in order to compare class data. Students were able to view the class averages, not student averages, to determine their increased abilities in *Thinking with Mathematical Models.*

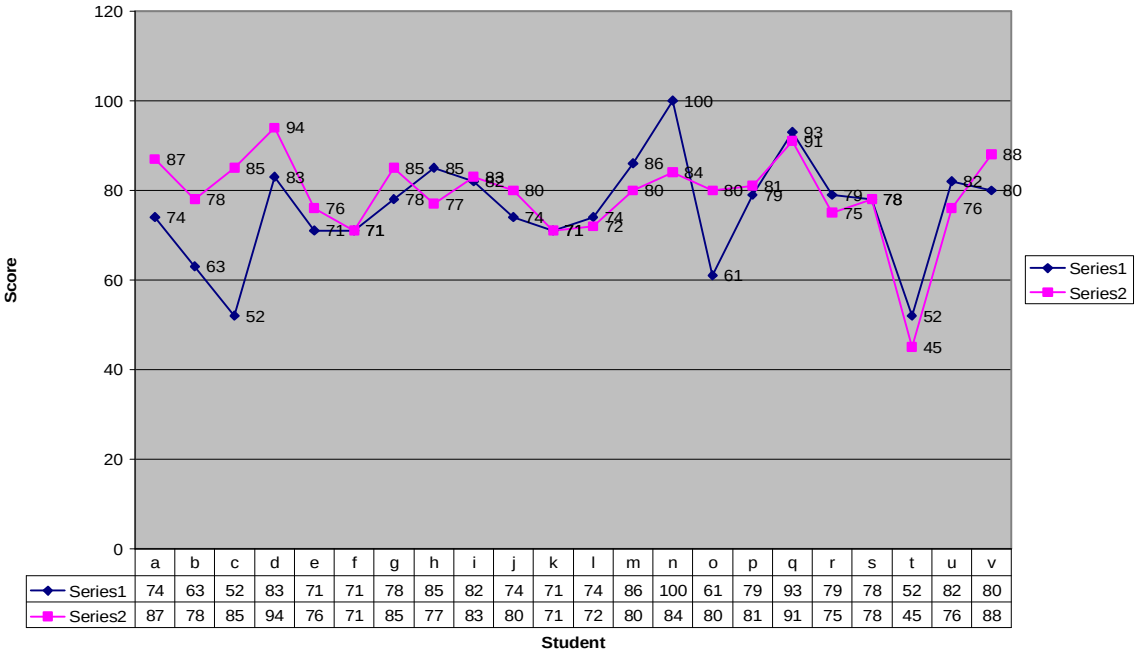
Period 2 - Thinking with Mathematical Models



	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v
Series1	44	82	73	80	90	76	73	99	81	102	60	90	73	70	55	61	81	71	80	71	91	49
Series2	96	83	71	86	92	94	97	94	100	104	59	99	80	61	81	60	83	82	81	88	83	92

Student

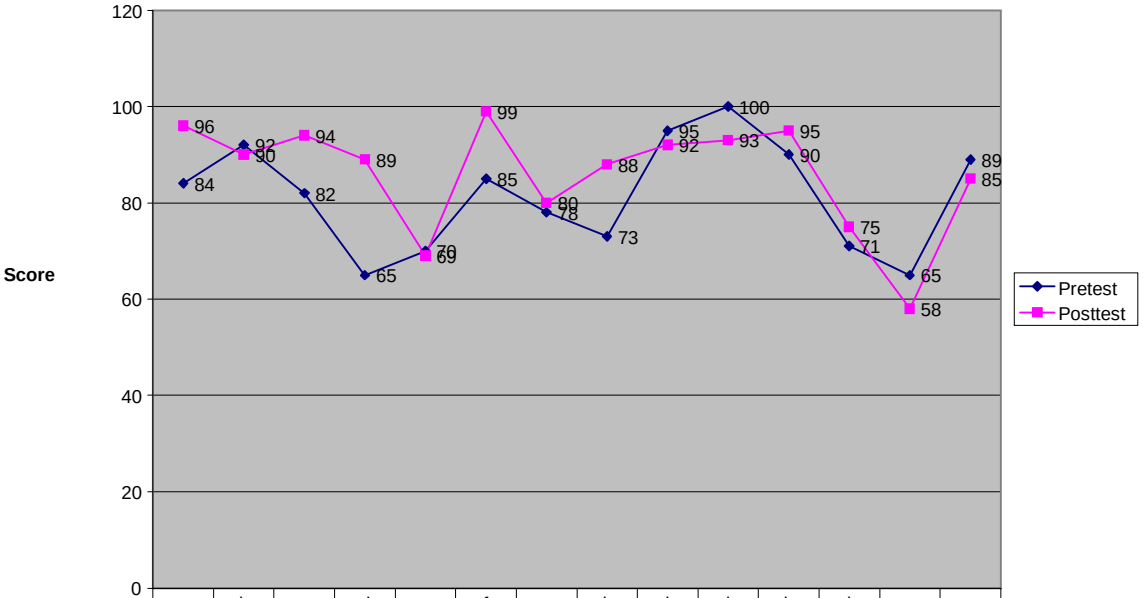
Period 4 - Thinking with Mathematical Models



	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v
Series1	74	63	52	83	71	71	78	85	82	74	71	74	86	100	61	79	93	79	78	52	82	80
Series2	87	78	85	94	76	71	85	77	83	80	71	72	80	84	80	81	91	75	78	45	76	88

Student

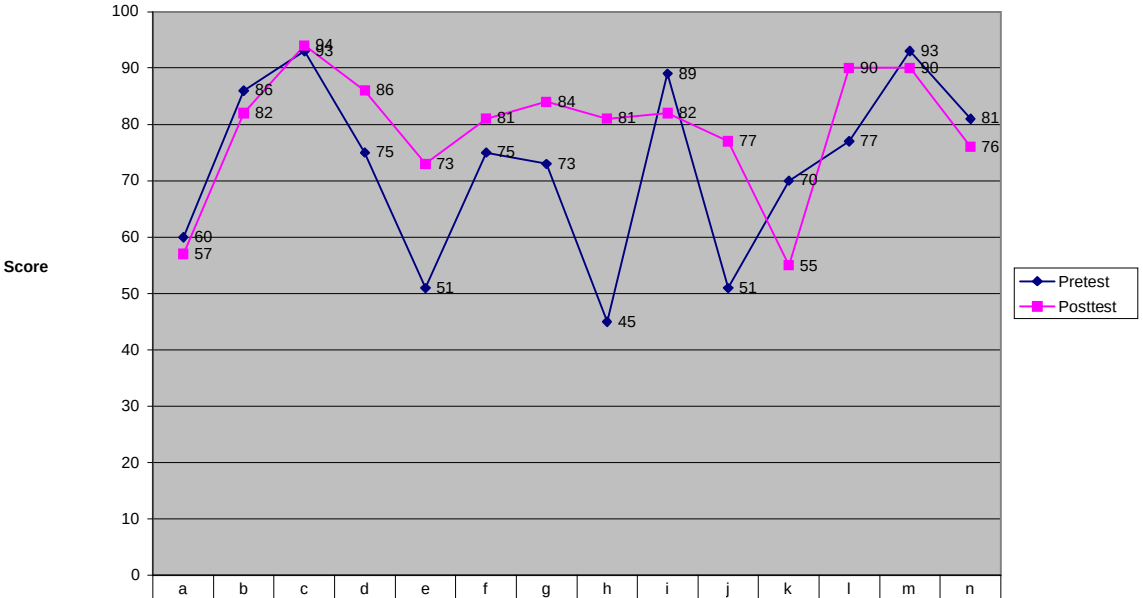
Period 6 - Thinking with Mathematical Models



	a	b	c	d	e	f	g	h	i	j	k	l	m	n
Pretest	84	92	82	65	70	85	78	73	95	100	90	71	65	89
Posttest	96	90	94	89	69	99	80	88	92	93	95	75	58	85

Student

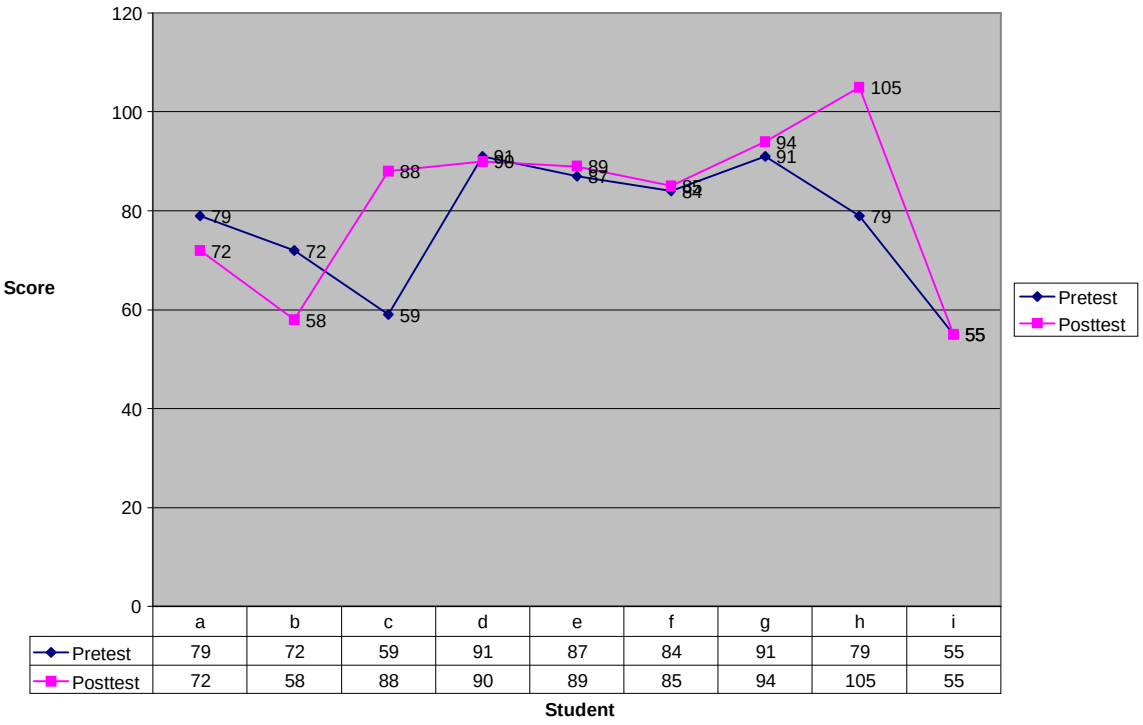
Period 8 - Thinking with Mathematical Models



	a	b	c	d	e	f	g	h	i	j	k	l	m	n
Pretest	60	86	93	75	51	75	73	45	89	51	70	77	93	81
Posttest	57	82	94	86	73	81	84	81	82	77	55	90	90	76

Student

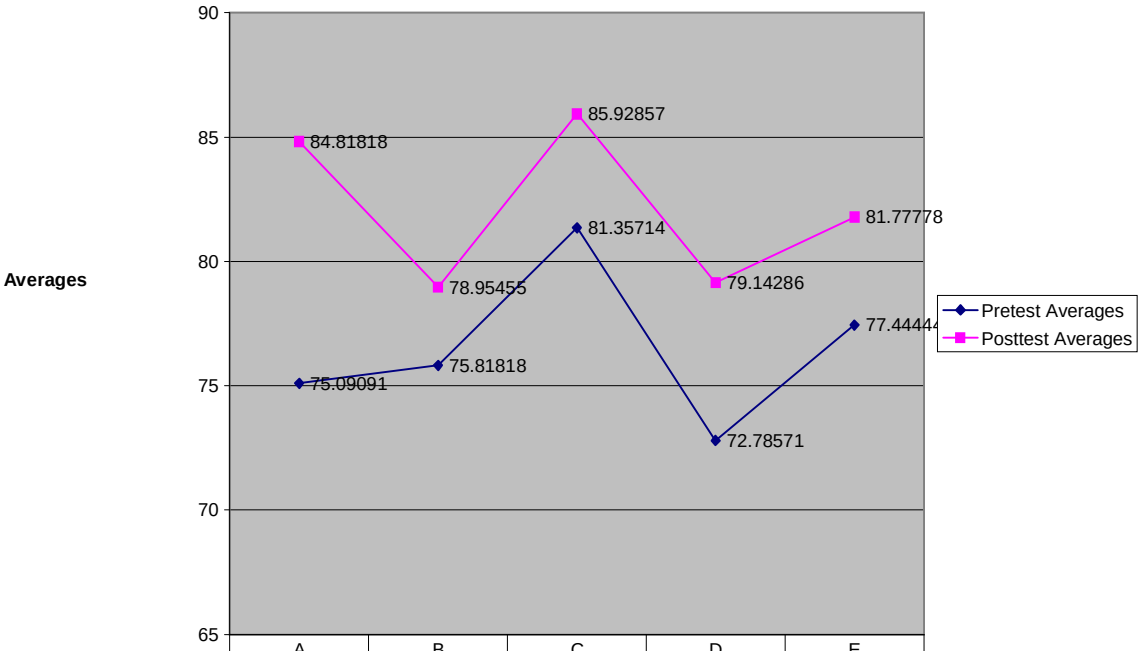
Period 9 - Thinking with Mathematical Models



	a	b	c	d	e	f	g	h	i
Pretest	79	72	59	91	87	84	91	79	55
Posttest	72	58	88	90	89	85	94	105	55

Student

Thinking with Mathematical Models



	A	B	C	D	E
Pretest Averages	75.09091	75.81818	81.35714	72.78571	77.44444
Posttest Averages	84.81818	78.95455	85.92857	79.14286	81.77778

Class

To conclude our study of graphing, students worked on the third unit, called *Growing, Growing, Growing.* This unit was based on exponential relationships and students reinforced their understanding of nonlinear relationships. To begin, students cut ballots for a school election. Each student was given only one sheet of paper and created as many ballots as possible that were the same size. Students recorded the number of cuts they made and the number of ballots that resulted. When students made only one cut, they created 2 ballots, but when they made five cuts, they created 32 ballots. Students compared their results with one another and found that the number of ballots doubled from the previous cut. Later, they discovered that this "doubling" was an exponential growth with a base of 2 and a changing exponent that depended on the number of cuts. Students were able to plot their points from their table in TI Interactive computer software to create a nonlinear relationship.

After cutting ballots, students moved on to an activity in which they were to repay a peasant for saving the princess of the kingdom of Montarek. Using a chessboard, the peasant was given several choices for rewards. The peasant was then to choose the reward that he felt adequately repaid him for saving the princess. Students were split into groups and each group was given one reward. The groups tried to convince the peasant that their reward was the best. Students presented their rewards using PowerPoint® presentation graphics program, Excel spreadsheet software, and TI interactive computer software to the class.

Group one's reward was to place one coin on the first square of the chessboard, two on the second square, four on the third square, eight on the fourth and so on, doubling the number of coins from the previous square until they reached the sixty-fourth square on the chessboard. Group two's reward was to make a chessboard with only sixteen squares but place one coin on the first square, three on the second square, nine on the third square, and so on, tripling the number of coins from the previous square until they reached the sixteenth square. Group three's reward was to make a chessboard with only twelve squares but place one coin on the first square, four on the second square, sixteen on the third square, and so on, increasing by four times the previous amount until they reached the twelfth square. Group four's reward was to create a chessboard with sixteen squares but place 500 coins on the first square, 1000 on the second square, 2000 on the third square, and so on, doubling previous amount until they reached the sixteenth square. Group

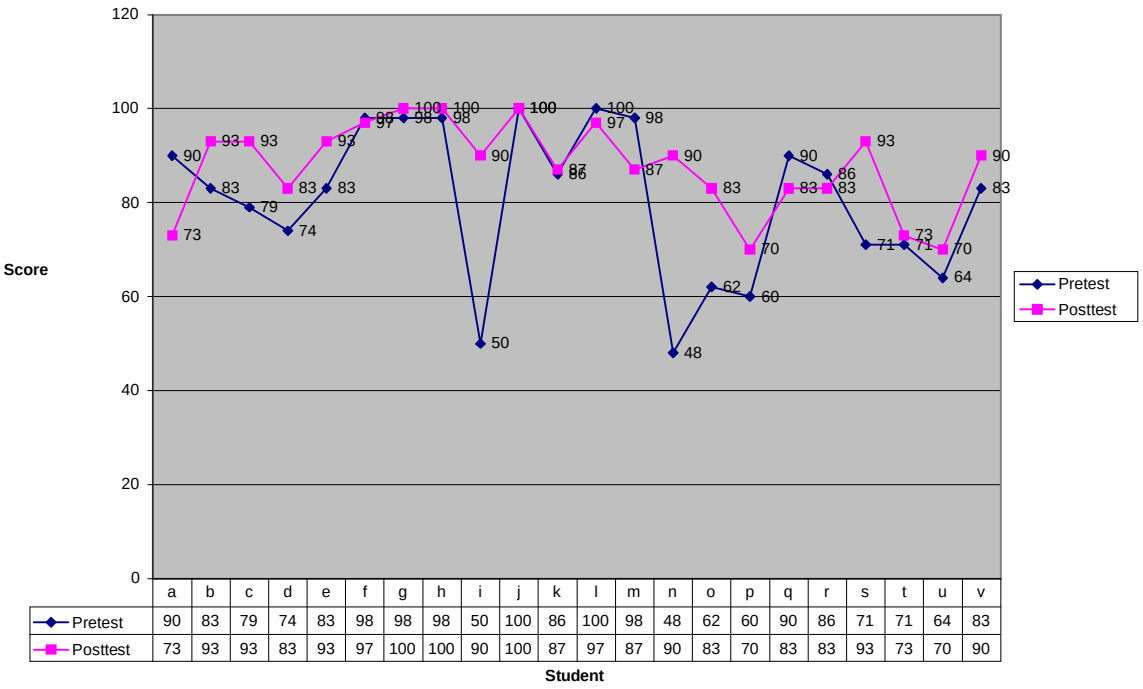
five's reward was to create a chessboard with sixteen squares but place 5 million coins on the first square, 6 million coins on the second square, 7 million coins on the third square, and so on, adding one million coins until they reached the sixteenth square.

Students did their best to convince their classmates to choose their rewards though no one was sure until the last presentation which reward was the best. Each class did conclude that Group one's reward was the best reward by far. All groups displayed tables and graphs of values for the number of coins that were on each square of their chessboard. Then, we created equations to illustrate the situation. From the graphs, tables, and equations, students found the properties that created linear relationships, nonlinear relationships, and exponential relationships.

We concluded the unit of exponential relationships by studying the rabbit growth in Australia in 1859. Students researched the experiment using the Internet and found that twenty-four rabbits were introduced in 1859 and that there were more than 600 million rabbits within a century. We estimated the number of rabbits that the twenty-four rabbits may have reproduced in one year if twelve of those rabbits were male and twelve were females. Students found on the Internet that the gestational period for rabbits is only thirty-one days and that they could have anywhere from two to twelve rabbits. We averaged that a female could have about six litters a year and an average of seven rabbits per litter. So, by the end of the first year, we determined that there may have been about 22,665 rabbits in Australia. We then used that information to determine the number of rabbits for the second, third and fourth years. Students plotted the data on a graph to see that the number of rabbits increased exponentially.

When comparing the results for the pretest and the posttest for *Growing, Growing, Growing,* the task was more difficult. The pretest consisted of both multiple-choice and open-ended questions but the posttest was based on a project. Students used their understanding of exponents and exponential relationships to determine the amount of bacteria that forms in your mouth between brushings. Students demonstrated a more consistent understanding of exponents in the posttest then in the pretest. By looking at the results, you can see that the range of scores on the posttest is smaller than the pretest in most classes. The average increase of scores, using the SMART Board interactive whiteboard for *Growing, Growing, Growing,* was 4.864502.

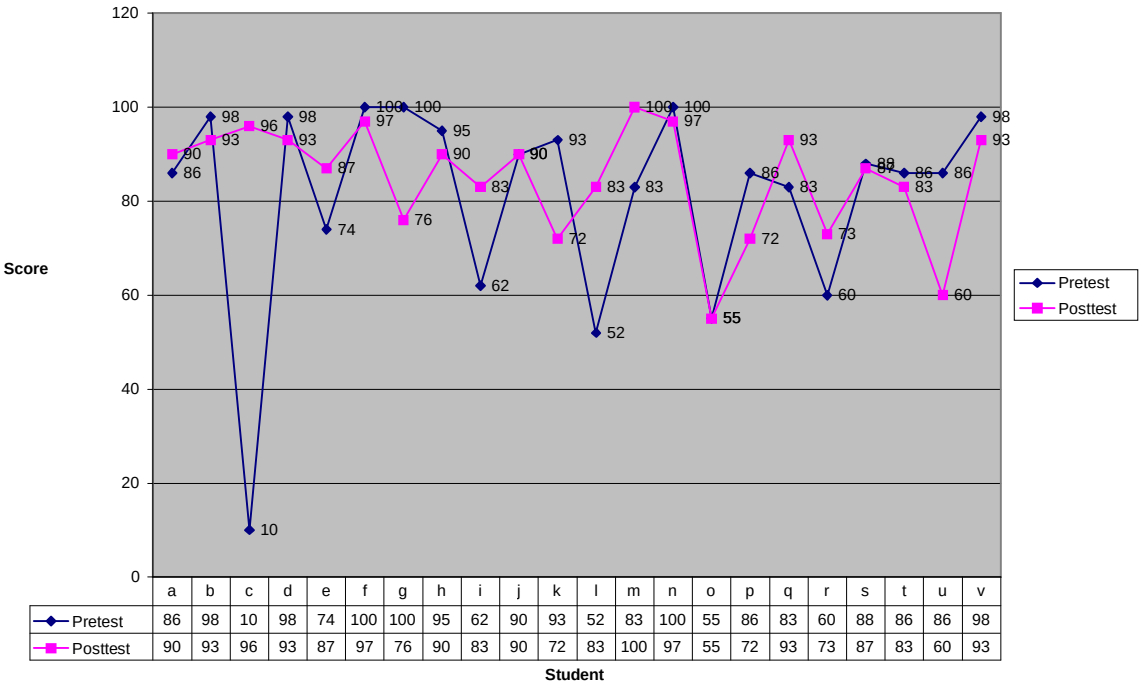
Period 2 - Growing, Growing, Growing



	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v
Pretest	90	83	79	74	83	98	98	98	50	100	86	100	98	48	62	60	90	86	71	71	64	83
Posttest	73	93	93	83	93	97	100	100	90	100	87	97	87	90	83	70	83	83	93	73	70	90

Student

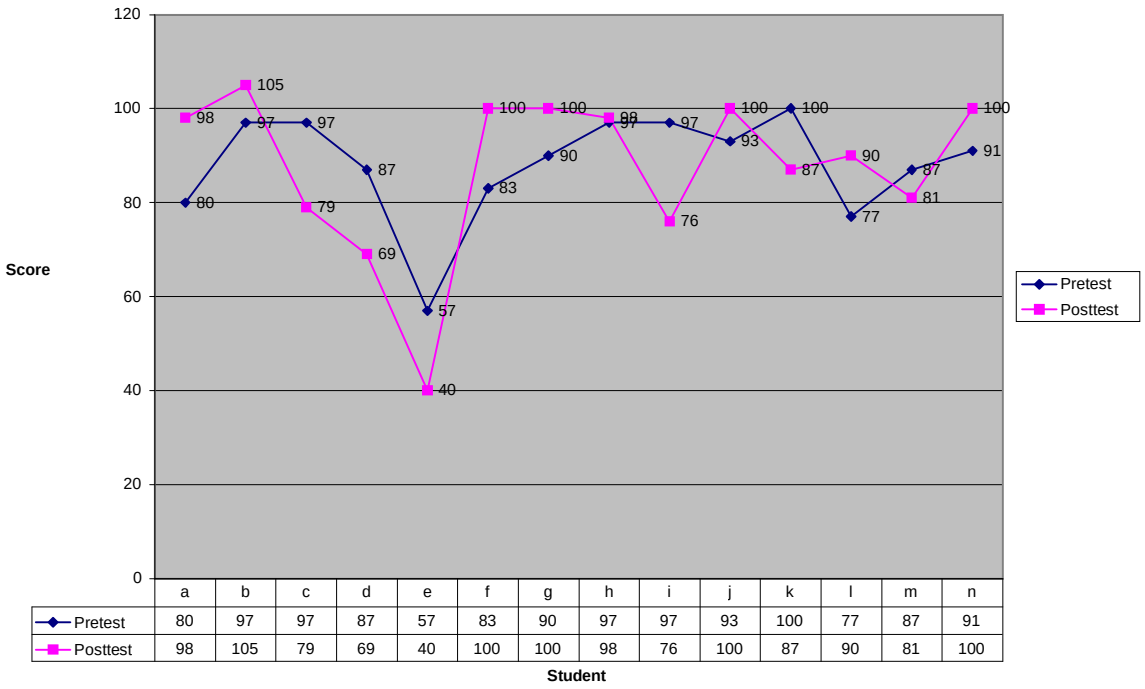
Period 4 - Growing, Growing, Growing



	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v
Pretest	86	98	10	98	74	100	100	95	62	90	93	52	83	100	55	86	83	60	88	86	86	98
Posttest	90	93	96	93	87	97	76	90	83	90	72	83	100	97	55	72	93	73	87	83	60	93

Student

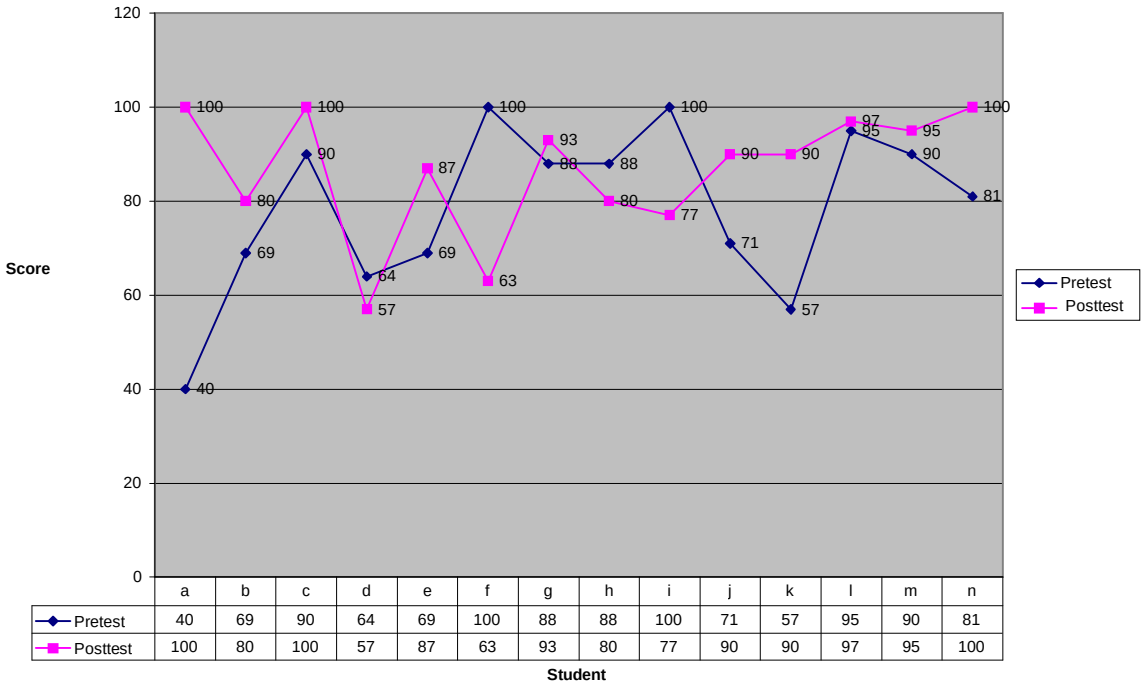
Period 6 - Growing, Growing, Growing



	a	b	c	d	e	f	g	h	i	j	k	l	m	n
Pretest	80	97	97	87	57	83	90	97	97	93	100	77	87	91
Posttest	98	105	79	69	40	100	100	98	76	100	87	90	81	100

Student

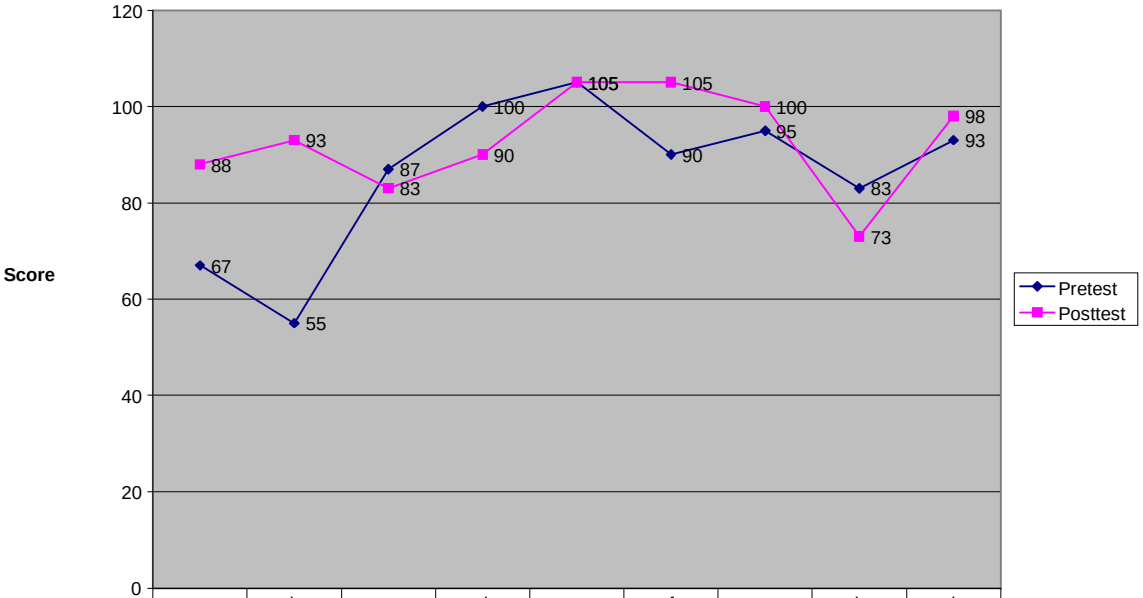
Period 8 - Growing, Growing, Growing



	a	b	c	d	e	f	g	h	i	j	k	l	m	n
Pretest	40	69	90	64	69	100	88	88	100	71	57	95	90	81
Posttest	100	80	100	57	87	63	93	80	77	90	90	97	95	100

Student

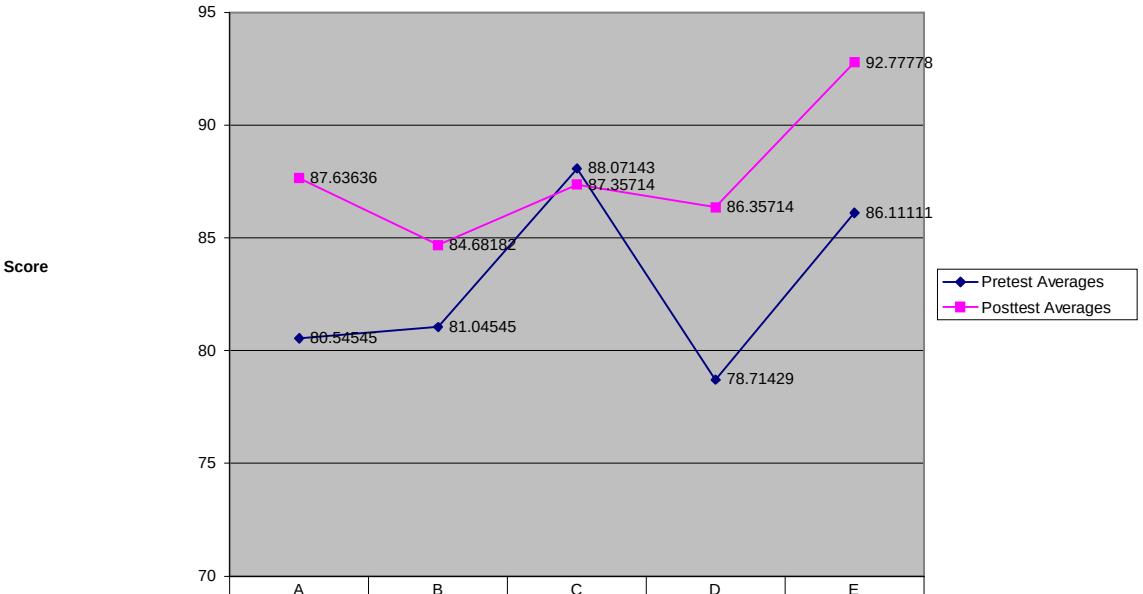
Period 9 - Growing, Growing, Growing



	a	b	c	d	e	f	g	h	i
Pretest	67	55	87	100	105	90	95	83	93
Posttest	88	93	83	90	105	105	100	73	98

Student

Growing, Growing, Growing



	A	B	C	D	E
Pretest Averages	80.54545	81.04545	88.07143	78.71429	86.11111
Posttest Averages	87.63636	84.68182	87.35714	86.35714	92.77778

Class

When conducting experiments, taking notes, creating graphs and composing tables, the SMART Board interactive whiteboard allowed the eighth-graders at Woodstown Middle School to use a hands-on approach to understanding linear, nonlinear and exponential relationships. This opportunity to use the SMART Board interactive whiteboards increased the creativity and alertness of the students in my class. Each day, we took time to explore the functions and abilities that the SMART Board interactive whiteboard gave to us. I also found that the students are more involved when they have the opportunity to visually show their work and compare it to the work of other students in their class, as well as the work of students in other classes. Although neither I, nor the students, were experts at using the SMART Board interactive whiteboard, we worked together to learn the capabilities that it allowed us to have in the classroom and more specifically with graphing. The SMART Board interactive whiteboard opened many doors for both myself, as an educator, and for my students, as learners.

References:

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